C++ Review DUNE 💻

Una organización donde compartir notas acerca de C++ con PDFs escritos en LaTeX.

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📃 Pad de apuntes

📎 Liga del PDF

🎬 Sesión grabada en `diode.zone`

Vector space

A vector space is a set V along with an addition on V and a scalar multiplication on V such that the following properties hold:

- $u + w = w + u$ for all $u, w \in V$;
- $(u + v) + w = u + (v + w)$ and $(ab)\,u = a\,(bu)$ for all $u, v, w \in V$ and all $a, b \in \mathbb{F}$;
- there exists $0 \in V$ such that $u + 0 = u$ for all $u \in V$;
- for every $u \in V$, there exists $w \in V$ such that $u + w = 0$;
- $1u = u$ for all $u \in V$;
- $a\,(u + w) = au + aw$ and $(a + b)\,u = au + bu$ for all $a, b \in \mathbb{F}$ and all $u, w \in V$.

Example

The set

$$\{f : [0, 1] \to \mathbb{R} \mid f \text{ is continuous.}\}$$

is a vector space with the operations:

1. $(f + g)\,(x) = (f)\,(x) + (g)\,(x)$.
2. $(cf)\,(x) = cf\,(x)$, for $c \in \mathbb{F}$.

Hamel basis

A basis of V is a subset H or list of vectors in V that is linearly independent and spans V.

1. $\forall F \subseteq H$, F is finite, $\sum_{f \in F} \lambda_f \cdot f = 0_V$, with $\lambda_f \in \mathbb{F} \implies \lambda_f = 0_{\mathbb{F}}$.
2. $\forall u \in V$, $\exists F$ finite, $F \subseteq H$ such that $\sum_{f \in F} \lambda_f \cdot f = u$, with $\lambda_f \in \mathbb{F}$.

Remark

The vector space $\{f : [0,1] \to \mathbb{R} \mid f \text{ is continuous.}\}$ has uncountable Hamel dimension.

Norm

A norm on V is a map from V to the nonnegative reals:

$$\|\cdot\| : V \to \mathbb{R}_{\geq 0}$$

satisfying

- $\forall x \in V : \|x\| = 0 \iff x = 0_V$.
- $\forall x \in V, \lambda \in \mathbb{F} : \|\lambda x\| = |\lambda| \, \|x\|$.
- $\forall x, y \in V : \|x + y\| \leq \|x\| + \|y\|$.

Normed vector space

Let V be a vector space over $\mathbb{F}$. Let $\|\cdot\|$ be a norm on V. Then, $(V, \|\cdot\|)$ is a normed vector space.

Let V_i, $i = 1, \ldots, n$, be a normed vector space of dimension n_i with a scalar product, we define

$$\prod_{i=1}^{n} V_i \coloneqq \{(v_1, \ldots, v_n) \mid v_1 \in V_1, \ldots, v_n \in V_n\}$$

is a vector space of dimension $\sum_{i=1}^{n} n_i$.

Field vector

```
template< class K, int SIZE >
class FieldVector;
```

represent a low-dimensional vector space with in $V = \mathbb{F}^n$ over the field $\mathbb{F}$.

```
Dune::FieldVector<double, 3> v;
```

Block vector

```cpp
template<class B, class A=std::allocator<B> >
class BlockVector;
```

```cpp
Dune::BlockVector<Dune::FieldVector<double, 3> >
```

Linear map

Let V and W two vector spaces. A linear map from V to W is a function $T : V \to W$ with the following properties:

- $T(u_1 + u_2) = T(u_1) + T(u_2)$ for all $u_1, u_2 \in V$;
- $T(\lambda u) = \lambda(Tu)$ for all $\lambda \in \mathbb{F}$ and all $u \in V$.

Example

Backward shift: Define $T \in \mathcal{L}(\mathbb{F}^\infty, \mathbb{F}^\infty)$ by

$$T(x_1, \ldots) = (x_2, \ldots).$$

Matrix of a linear map

Suppose $T \in \mathcal{L}(V, W)$.